MUTUAL ATTRACTION GUIDED SEARCH: A NOVEL SOLUTION METHOD FOR THE CURVATURE CONSTRAINED TRAVELING SALESMAN PROBLEM

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ABSTRACT
TSP solution techniques are extensively used for vehicle route planning and are NP-complete even in the most basic form; however, the problem complexity is even greater for vehicles with a limited turning radius. Mutual Attraction Guided Search (MAGS), a new algorithm in development by the PI, quickly converges on local optima to the curvature constrained TSP. The basic implementation of MAGS does not have a mechanism for escaping local minima. Ongoing development is in progress to develop a hybrid algorithm, using EA techniques to combine and mutate good solutions, facilitating global optimization. The hybrid alternates between a stochastic evolutionary search using EA operators, and a deterministic local search using MAGS in order to achieve an appropriate balance between global and local search.

1. INTRODUCTION
Path planning for automated vehicles is a challenging task and is the subject of much research interest, especially as automation has been increasingly integrated into all parts of society including navigation and even vehicle piloting. Traditional Euclidean TSP has been widely researched as a path planning algorithm and is known in itself to be NP-hard. [1] Euclidean TSP can be used for route planning situations when targets are spaced far enough that the turning radius of the vehicle is negligible; however, if a cluster of targets are spaced on a similar scale as the vehicle's turning radius, the curvature constraints must be considered when planning the path.

Optimizing a curvature constrained TSP (CCTSP) path is much more complex than many TSP variants since any change in the angle of travel through any node changes the optimal path to all other nodes. This is illustrated in Fig. 1 for the trivial case of a two node problem in which the turning radius is limited to 1 unit and the nodes have a separation of two units. Figure 1 (b) depicts the optimal solution. Figure 1. (a) shows that even modifying only one node orientation can significantly affect the path length between nodes. The curvature constrained TSP therefore cannot be reduced to a problem on a finite-dimensional graph as can many other TSP variants. [2] Thus, common combinatorial optimization tools cannot be easily applied. Research in this area is actively making improvements in solution quality; however, the complexity of the problem and more limited application scope have resulted in less research attention than simpler TSP variants even though the practical application possibilities have great significance.

The PI has developed a novel solution method (MAGS) that uses an iterative process to simultaneously optimize both angle of travel through each target as well as the ordering of the targets. MAGS has the ability to deterministically locate a locally optimum solution quickly and can be easily adjusted to optimize for the acceleration requirements of a specific vehicle; however, because of the deterministic nature of MAGS, the solution achieved is dictated by the starting configuration and can be easily trapped in local minima. To complement the local search abilities of MAGS, an EA hybridization is proposed. The development of appropriate evolutionary operators can be used to mutate and recombine solutions found by MAGS. The new generation produced by the evolutionary process can then be matured with MAGS before applying the evolutionary operators again. It is expected that this hybridization will provide the necessary balance between local and global search that is required to locate a globally optimal solution.

2. RELATED WORK
The TSP has been a popular research subject and is widely used in route planning for automated vehicles. The TSP was only designed to address the ordering of the target nodes and does not consider the actual path necessary to move between points. Due to physical limitations, no vehicle is able to make radical changes in direction instantaneously. In many cases, particularly aerial vehicles, the dominating factor that restricts the radius of a turn is the forward momentum of the vehicle. A constant maximum normal force results in a radius bound that
is proportional to the square of the velocity. In cases when the target waypoints are widely separated in comparison to the radius limit, the Euclidean TSP can still be used to plan the basic path, with adjustments to take into account the turning radius. However, when the waypoint density is comparable to the turning radius, the ordering of the targets as given by Euclidean TSP becomes sub-optimal.[3] Because of the exponential velocity relationship, increasing speed requires either handling much larger forces, or a larger turning radius. For a military surveillance vehicle, speed can be essential in order to avoid enemy fire; therefore it is not desirable to limit speed in order to make sharp turns.

Current applicable research is primarily focused on the Dubins TSP variant. Dubins first described the minimum path distance between points with prescribed initial and terminal tangent angles.[4] Dubins curves form a path that alternates between straight-line segments and circular curves with the minimum turning radius. These curves are optimal when a vehicle's turning radius has a constant limit, either because the dominating factor is a physical constraint such as the steering mechanism, or because the dominant factor is the momentum and the vehicle maintains a constant speed. In order to deal with the additional complexity of the curvature constraints, current solutions either try to optimize node ordering and angle of travel separately -- sometimes alternating between the optimization processes -- or use an evolutionary process to make small changes to both node ordering and angle of travel. No research has been published using a solution method that optimizes both travel angles and node ordering simultaneously. The research using the Dubins TSP variant is also sub-optimal in many cases for vehicles whose minimum turning radius is dependent on a variable speed. Tighter curves require a reduced speed, therefore Dubins solutions, while producing the shortest path, may require more time to traverse the path since the vehicle must travel at a slower speed. A better solution would give preference to shallow curves when possible without significantly increasing the path length in order to allow the vehicle to traverse the path more quickly.

3. METHODOLOGY

The inspiration for MAGS lies in the interaction of bar magnets through attractive and repulsive fields. Each target node is modeled with a vector field similar to a bar magnet as shown in Fig. 2, though very simplistic in comparison to actual magnetic fields. The orientation of the node represents the angle at which the vehicle will pass through the node, with the vehicle passing from ‘south’ to the ‘north’ in the magnet illustration.

A simplistic version of the solution process can then be conceptualized as if each node of the TSP was represented by a bar magnet centered at the node location and free to rotate about its center. If a set of magnets were thus arranged, they would each attempt to align themselves with the net field produced at their location by the other nodes. As the nodes rotate, the field adjusts accordingly until locally optimal magnetic loops are formed. MAGS utilizes a similar interaction in order to form path loops through the targets based on the vector field.

The field strength produced by each node at a given location varies with respect to the relative position and the orientation of the node. Figure 3 depicts a node at an angle of alpha – represented by the blue arrow. The field strength – represented by the red arrow – is desired at some arbitrary position. The field strength at the desired location is calculated based on the properties of a circular arc connecting the node with the point of interest and tangent to the node orientation. In order to minimize travel time along the path, path length must be minimized and the radius of curvature maximized. Therefore the magnitude of the field produced at the point of interest is based on the arc length $C$ and the radius $R$ as shown in Fig. 3. The orientation of the field produced at the point of interest is tangent to the arc and is given by beta in Fig. 3.

![Figure 3: Field Strength and Angle Contribution](image)

The distance and angle between nodes (given by $E$ and theta in Fig. 3) are constant for a given problem and can be calculated once at the beginning of the algorithm. The basic solution process then consists of iteratively repeating the following steps until convergence:

1. Calculate the magnitude and direction of the net field produced at each node based on the current orientation of the other nodes.
2. Update the orientation of each node by adjusting it by some percentage of the difference between the current
Several considerations must be taken into account which add some further complexity to the procedure. In general, the magnitude of the field produced is directly proportional to the radius of the arc between two nodes. When the orientation of one node is directly in line with another node however, an infinite radius results, therefore some maximum radius limit must be enforced before calculating the magnitude. A minimum radius limit may often also be enforced unless the vehicle is capable of a zero-radius turn. In the case of a minimum radius limit, the field magnitude is simply set to zero, while for the maximum radius limit, the radius is set to the maximum radius value and the arc distance is set to the Euclidian distance between the nodes.

Additionally, the simple form of the algorithm only provides a solution in the form of node angles and a vector field between nodes. In order to quickly generate a path, it is necessary to also identify a node ordering. To accomplish this, a weight matrix representing the connectivity between nodes is used. At the beginning of the algorithm, all connection weights are set to 100%. After every iteration, a relative ranking of other nodes is generated for each node by scaling the magnitude of the individual field contributions based on the largest value. The connection between each pair of nodes is then updated based on the product of their rankings of each other. A connection adjustment rate can be used to tune the loyalty of the nodes. These connection weights are applied to the field magnitude contributions from each node in the next iteration when calculating the net field at each node. For each pair of nodes, there are also two connectivity weights to account for the direction of travel between the nodes (represented by the red and blue line segments in Fig. 1). Once the algorithm converges, the ordering of the nodes can be traced by identifying the highest ranking connection from each node.

4. CURRENT RESULTS AND FUTURE WORK

Further testing of the MAGS framework is necessary to determine what characteristics define problems that are better solved with MAGS rather than traditional Dubins vehicle methods. If simple measures such as target point cluster density are found to provide a reasonable distinction, a large target set could be divided into sub-problems such that dense clusters can be isolated and solved with MAGS, while other portions of the path are found using simpler Dubins vehicle methods.

Further investigation into parameter tuning and alternate methods of merging multiple loops could also provide a way to improve performance.

Ongoing work towards the completion of the EA-MAGS hybrid should also greatly enhance the ability of MAGS escape local minima improving its performance. The level of improvement achieved by the hybrid will need to be compared to the increase in computational complexity in order to determine whether the benefits of the hybridization are practical for use in real-time path planning.

5. CONCLUSIONS

Current published methods for solving curvature constrained TSP problems are primarily limited to planning for constant speed travel using the Dubins vehicle model. These methods have been shown to work well for widely spaced targets, and for vehicles that travel with constant speed. However when targets are clustered more densely, these paths become increasingly sub-optimal for vehicles that have the ability to vary their speed. MAGS, the magnetics inspired algorithm developed by the PI has shown promise in quickly converging on a local optimum solution. The solution is based on a balance between minimizing path length and curvature in order to optimize path traversal time.

The primary drawback to MAGS is that it locates a solution deterministically based on the initial configuration. This causes it to be easily trapped by local minima. An evolutionary algorithm hybridization is in progress in order to alleviate this weakness.

8. ACKNOWLEDGMENTS

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9. REFERENCES