A NEW HEURISTIC FOR INVENTORY ROUTING PROBLEM WITH BACKLOG

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ABSTRACT
Inventory routing problem (IRP) is a major concern in operation management of a supply chain. The aim is to integrate the transportation activities and inventory management along the supply chain and avoid inefficiencies caused by solving the underlying vehicle routing and inventory sub-problems separately. This paper can be considered as an extension of single depot IRP with order backlogs to a multi-depot case. We develop a mixed integer mathematical model for multi-depot inventory routing problems allowing order backlogs. The mathematical model is used to find the best compromise among transportation, holding and backlog costs for small instances. A novel method based on Simulated Annealing (SA) is proposed to solve larger IRPs with multiple depots. Computational results for single depot problems are compared with those reported in a recent article with similar assumptions. We have extended the proposed SA algorithm for solving multi-depot IRPs with the possibility of order backlogging. Since we could not find any reported computational results for this type of multi-depot problems; we have compared the results with exact solutions obtained by Cplex solver.

1. INTRODUCTION
In recent years, one of the most interesting and challenging issues in the area of supply chain management is the integration and coordination of activities. Inventory routing problem (IRP) is one of the famous issues in this category. [1] Introduced this problem as a combination of Vehicle Routing Problem (VRP) and inventory control and showed that it is NP-hard. Another interpretation of IRP was given in [2], as determination of the set of customers in a supply network with specific demands to be served in each day. A more precise definition, as suggested by [3], states that an IRP is how to distribute a single product from a single supplier to a set of geographically scattered retailers/customers by a fleet of homogenous vehicles over a period of time [4]. The retailers store specific amount of inventory that is consumed by a certain rate. The objective is to find the minimum total distribution and inventory holding cost while assuring customers’ demands are satisfied in due time. Other constraints have been added to the original problem over time that led to various types of IRPs.

There exist various applications of this class of problems reported in the literature; for instance, distributing liquid propane in petrochemical industry [5], retail products in Netherland [6], automobile components [7] and frozen products [8]. In maritime, the distribution of ammonia from a central depot to a group of consumers has been reported in [9].

According to a recent survey in [9], there exists very few reported research on inventory routing problem with backlog (IRPB). [10] Proposed a mixed integer mathematical model for a single period IRP with possibility of backlog and deterministic demands in which they assumed that a central depot would serve a set of customers using heterogeneous vehicles. Later, a single-depot multi-period IRPB was put forward by [11] while the demand rates were supposed to be deterministic. The solution method used therein was based on decomposition of the problem into ordering and distributing sub-problems. [12, 13] investigated IRPB with infinite time horizon and stochastic demands. They found a lower bound for cost of serving the customers by unlimited number of homogenous vehicles in a direct shipping format. [14] Investigated a single depot IRPB with stochastic demand. They assumed a single vehicle would serve the retailers in direct and multiple formats and their solution approach was Monte-Carlo simulation and heavy traffic analysis. [15] Put forward an IRPB with stochastic demand in which a single vehicle serves the retailers. They proposed an analytical model and solved it using simulation and a change-revert heuristic method. Other researchers investigated the many-to-many topology in IRP with finite time horizon [16-19]. [20] Introduced a single depot IRPB with deterministic demand allowing multiple trips taken by a fleet of heterogeneous vehicles. They introduced a genetic-based constructive heuristic method for solving the problem. Recently, they proposed another constructive heuristic method for the same problem and improved their solution [21].

Multi-depot IRPs have been investigated in several cases before but without considering the possibility of order backlogs [9]. This paper can be considered as an extension of single depot IRP with order backlogs to a multi-depot problem. We develop a mixed integer mathematical model for multi-depot inventory routing problems allowing order backlogs. The mathematical model is used to find the best compromise among transportation, holding and backlog costs for small instances. We extend and combine the best existing heuristics for solving the vehicle routing and inventory routing sub-problems and improve their solutions by applying the algorithms iteratively. This allows one to solve larger problems and get solutions closer to optimality in reasonable time. Furthermore, an efficient simulated annealing algorithm has been developed and used for solving medium to large instances.

The rest of this paper is organized as follows. In Section 2, a mathematical model is developed for the problem. The solution algorithm is described in Section 3, followed by the computational results in Section 4. Conclusions are in Section 5.
2. PROBLEM DEFINITION AND THE MATHEMATICAL MODEL

We consider a two echelon supply chain in which a set of retailers are served by a fleet of capacitated homogenous vehicles. In the first case, a central depot (supplier) serves a set of geographically scattered retailers having certain demands. Retailers have limited storage capacities and cannot accommodate as much supply as they want. The problem is a multi-period planning with a finite time horizon. Retailers’ demands in each period are assumed to be known and should be met by the end of the designated time period. The suppliers incur a penalty named backlog cost if the delivery is late. Retailers who encountered order backlogs shall get service in later periods. The objective is to find the best trade-off among transportation, backlog and holding cost. In the multi-depot case, the central depot is replaced with several depots. All other assumptions remain unchanged. The assumptions on the VRP part in both cases are the same as in the classical models.

In this section, we develop a mixed integer mathematical model for Multi Depot Inventory Routing Problem with Backlog (MDIRPB). First, the sets, constants, parameters and decision variables used in the model are introduced.

Sets:
- \( R \): retailer set
- \( D \): depot set
- \( T \): time period set
- \( K \): vehicle set

Constants and Parameters:
- \( c_{ij} \): Travel cost from retailer/depot \( i \) to retailer/depot \( j \) for \( \forall i,j \in (R \cup D) \)
- \( \pi_i \): Backlog cost per period per unit for \( \forall i \in R \)
- \( h_i \): Holding cost per period per unit for \( \forall i \in R \)
- \( d_{it} \): Demand of retailer \( i \) in period \( t \) for \( \forall i \in R \cup D \) and \( \forall t \in T \)
- \( B_c \): Travel capacity of retailer \( i \) for \( \forall i \in R \)
- \( s_{ij} \): Time to serve retailer \( i \) by vehicle \( k \) for \( \forall i \in R \) and \( \forall k \in K \)
- \( t_{ijl} \): travel time from retailer/supplier \( i \) to retailer/supplier \( j \) for \( \forall i,j \in (R \cup D) \)
- \( b_i \): Initial amount of inventory of retailer \( i \) for \( \forall i \in R \)
- \( L \): Length of each time period
- \( Q \): Vehicle capacity
- \( M \): a big positive constant
- \( N \): Maximum number of the retailers

Decision variables:

For \( \forall i, j \in (R \cup D) \) and \( \forall t \in T \) and \( \forall k \in K \),

\( X_{ijkt} \): Inventory amount in retailer \( i \) store in period \( t \) for \( \forall i \in R \) and \( \forall t \in T \)

\( I_{it}^+ \): Inventory amount in retailer \( i \) store in period \( t \) for \( \forall i \in R \) and \( \forall t \in T \)

\( W_{ikt} \): The amount delivered to retailer \( i \) by vehicle \( k \) in period \( t \) for \( \forall i \in R \) and \( \forall k \in K \) and \( \forall t \in T \)

\( U_{it} \): If a vehicle is serving retailer \( i \), the number of the other retailers that should be visited by that vehicle after customer \( i \) in each period of time. It should be noted that \( U_{it} \) is unique for \( \forall i \in R \) and \( \forall t \in T \)

The Mathematical Model is given by:

**Objective function:**

\[
\text{Min} \ z = \left( \sum_{i \in RU} \sum_{j \in RU} \sum_{k \in K} \sum_{t \in T} c_{ij}X_{ijkt} \right) + \left( \sum_{i \in R} \sum_{t \in T} \pi_i I_{it}^+ \right) + \left( \sum_{i \in R} \sum_{t \in T} \xi_i I_{it}^- \right)
\]

**Subject to:**

\[
\sum_{j \in RU} X_{ijkt} - \sum_{j \in RU} X_{njk} = 0 \quad \forall i \in R, \forall t \in T \quad (3)
\]

\[
\sum_{i \in R} \sum_{j \in RU} s_{ik} X_{ijkt} + \sum_{i \in R} \sum_{j \in RU} \sum_{j \neq j} t_{ij} X_{ijkt} \leq L \quad \forall k \in K, \forall t \in T \quad (4)
\]

\[
\sum_{i \in R \cup D} \sum_{j \in RU} s_{ik} X_{ijkt} + \sum_{i \in R \cup D} \sum_{j \in RU} \sum_{j \neq j} t_{ij} X_{ijkt} \leq L \quad \forall k \in K \quad (5)
\]

\[
\sum_{i \in R} \sum_{j \in RU} X_{ijkt} \leq M (U_{it} + 1) \quad \forall i \in R, \forall t \in T \quad (6)
\]

\[
\sum_{i \in R} \sum_{j \in RU} s_{ik} X_{ijkt} + \sum_{i \in R} \sum_{j \in RU} \sum_{j \neq j} t_{ij} X_{ijkt} \leq L \quad \forall k \in K \quad (7)
\]

\[
\sum_{i \in R \cup D} \sum_{j \in RU} s_{ik} X_{ijkt} + \sum_{i \in R \cup D} \sum_{j \in RU} \sum_{j \neq j} t_{ij} X_{ijkt} \leq L \quad \forall k \in K \quad (8)
\]

\[
W_{ikt} \leq Q \sum_{i \in R \cup D} X_{ijkt} \quad \forall j \in R, \forall k \in K, \forall t \in T \quad (9)
\]

\[
\forall k \in K, \forall t \in T \quad (10)
\]

\[
U_{it} = U_{j,t} + \sum_{k \in K} (N - 1) X_{ijkt} \leq (N - 1) - 1 \quad \forall i, j \in R \text{ } i \neq j \text{ } \forall t \in T \quad (11)
\]

\[
\forall i, j \in R \text{ } i \neq j \text{ } \forall t \in T \quad (12)
\]

\[
U_{it} \geq N - 1 \quad \forall i \in R, \forall t \in T \quad (13)
\]

\[
X_{ijkt} \in \{0, 1\} \quad \forall i, j \in R \cup D \text{ } i \neq j, \forall k \in K, \forall t \in T \quad (14)
\]

\[
W_{ikt} \geq 0 \quad \forall i \in R, \forall k \in K, \forall t \in T \quad (15)
\]

\[
I_{it}^+ \geq 0 \quad \forall i \in R, \forall t \in T \quad (16)
\]

\[
I_{it}^- \geq 0 \quad \forall i \in R, \forall t \in T \quad (17)
\]

\[
I_{it} \geq 0 \quad \forall i \in R, \forall t \in T \quad (18)
\]

The objective function Eq. (2) minimizes the total cost of transportation, backlog and holding inventories. The shortage and inventory carrying costs are calculated by the net backlog
3. THE SOLUTION METHOD

One of the key decision in inventory routing problem is to determine how much to deliver to each retailer in each period. The proposed heuristic method, named SA-UFT, is based on solving the underlying VRP component, the problem is NP-hard and its complexity grows exponentially by increasing the number of retailers, depots, time periods. The tighter the due dates and capacity limitations, the more difficult the problem becomes to solve.

3.1. MDVRP Solution Procedure

Vehicle routing problem is one of the key components of inventory routing problem. Therefore, the algorithm used for solving the underlying VRP should be an efficient one. In this paper, we employ an efficient algorithm for solving multi-depot vehicle routing problem. The method decomposes the multi-depot problem to several single depot ones following the ideas in [24]. Then, each VRP is solved using an efficient saving based algorithm of Clarke and Wright [25]. Afterwards, the solution is improved using a nearest neighborhood heuristic (NNH). The combination of these three algorithms helps us find a good solution for MDVRP part of MDIRPB.

3.2. SA-UFT Algorithm for Solving MDIRPB

We propose a simulated annealing (SA) based heuristic method, named SA-UFT, for solving MDVRP. Simulated annealing is a probabilistic method for finding the global minimum of a cost function that may have several local minima [26]. We have devised a heuristic rule for generating appropriate neighborhoods based on evaluating an unfitness function defined below. It involves the process of interchanging sets of retailers to be served in different periods of the planning horizon. All retailers are evaluated according to the unfitness function which shows how improperly retailers are assigned to be served in their designated periods. Then, a set of the retailers are selected to be removed from a specific period and assigned to another period so that their unfitness function is improved. The whole process of the solution method are explained in Fig.1. The Parameters and Notations used in the proposed SA-UFT neighborhood generation are as follows:

\[ T: \text{Number of planning periods} \]
\[ \text{iter}: \text{Iterations counter} \]
\[ UFT\_\text{changeamount}_{i,k,t}: \text{New delivery amount to customer } i \text{ in period } t \text{ by vehicle } k \]
\[ \text{changerate}: \text{The delivery probability} \]
\[ UFT\_\text{Value}_{i,t}: \text{Unfitness value for retailer } i \text{ in period } t \]
\[ \text{VRP}: \text{Vehicle routing cost} \]

**Neighborhood generation for each retailer:**

**Step1:** Get the initial solution \((W_{k,t})\), and evaluate \((Inv_{i,t})\).

**Step 2:** For each period \((t = 1: T)\) specify \((W_{k,t} > 0)\), and call them \((CW_t)\).

**Step 3:** For each period \((t = 1: T)\); Specify the retailers of \((CW_t)\); Sort those retailers in an ascendant order according to their travel cost;

**Step 4:** For each period \((t = 1: T)\);

\[ \text{if } inv_{i,t} > 0 \]

\[ UFT\_\text{changeamount}_{i,k,t} = 0; \]

\[ \text{else} \]

\[ UFT\_\text{changeamount}_{i,k,t} = \max (0, -Inv_{i,t}); \]

Set \(dt = 1;\)

While \((t + dt < T)\)

\[ \text{if } (\text{rand} > \text{changerate}) \]

\[ \text{if } (UFT\_\text{changeamount}_{i,k,t} + d_t + dt < Q) \]

\[ UFT\_\text{changeamount}_{i,k,t} = UFT\_\text{changeamount}_{i,k,t} + d_t + dt; \]

\[ \text{else} \]

Break;
Update $dt = dt + 1$;
\textit{end of loop}

\textbf{Step 5:} Calculate the unfitness value of each retailer for each period

$$UFT.Value_{i,t} = \max(0,inv_{i,t})\pi_i + \max(0,-inv_{i,t})\pi_i + \text{VRP}(W_{i,t_{\text{st}}}) - \max(0,inv_{i,t} - 2i - UFT.\text{change}_{i,t_{\text{st}}})\pi_i - \max(0,(-inv_{i,t} - W_{i,t_{\text{st}}}) + \text{UFT.\text{change}_{i,t_{\text{st}}}})\pi_i - \text{VRP}(UFT.\text{change}_{i,t_{\text{st}}})$$

\textbf{Step 6:} Sort $UFT.Value_{i,t}$ in a descendant order.

4. \textbf{COMPUTATIONAL RESULTS}

To test the efficiency of the proposed heuristic method, IRPB problems have been solved in two cases: single depot IRPB and multi depot IRPB. For the first case, we use [21] data set and compare SA-UFT costs with those of the ETCH-H and BDXH starting with ETCH-H algorithms of the mentioned article. For the second case, we generate random data, and benchmark the solution found by SA-UFT against the lower and upper bounds obtained by AMPL-CPLEX 10.1. The heuristic method has been coded and compiled in MATLAB (version R2008b) running under a Pentium dual-core processor with a clock speed of 1.67 GHz with 2GB RAM.

For single depot there are three scenarios proposed in [21]. The first and second scenarios contain 12 problem types. For each type, they generate and solve five samples. Each sample is shown by six digits. The first one represents the scenario number. The second and third ones are for customer count, and the fourth one is for time period count. The fifth one stands for the vehicle count, and the last one is the sample number in that problem type. Tables 1 shows some samples of results for the three scenarios. The results reported in Table 1 shows that SA-UFT makes a better trade-off between holding and transportation cost in almost all cases and leads to lower total costs for all the samples of ETCH-H. The run time for SA-UFT in all the scenarios is less than 5 seconds.

To evaluate SA-UFT efficiency for multi-depot IRPB, we generate test problems. Table 2 shows the results multi-depot IRPB. As can be seen in this table, the results obtained by SA-UFT are very close to those of Cplex. Furthermore, for displaying the impact of allowing order backlog, we compare the results of IRP with those of IRPB solved by the proposed algorithm and the results are in Table 3. We see that the results for IRPB are better than those of IRP for all the cases. Figure 2 shows the optimal routes for sample 1-0551-2. Figure 3 shows the convergence of SA-UFT in the total cost.

5. \textbf{CONCLUSION}

This paper investigates multi-depot inventory routing problems allowing order backlogs. Due to the inevitability of backlogging in real world problems, and the existence of more than one supplier depot in most cases, the combination of these two features in an integrated model is of practical importance. The method developed herein helps to decrease the cost in comparison with the case where single depot IRP with no possibility of shortage is considered. The resulting mixed integer model has been solved using a SA-based heuristic method proposed herein. The performance of the proposed heuristic is compared to that of ETCH-H appeared in a recent paper for single depot case [21]. The effectiveness of IRPB for
multi-depot case is verified by comparing the results with those obtained by Cplex solver. The numerical results show that SA-UFT outperforms existing ETCH-H in terms of cost minimization. Another comparison shows that the quality of solutions obtained by SA-UFT in multi-depot test problems is reasonably better than those found by Cplex. However, the main advantage of the proposed SA-UFT method is its ability to solve larger instances of multi-depot IRPB in a reasonable time.

Future research could be aimed at finding efficient lower bounds and an exact solution method for integrated inventory and routing problems as well as considering uncertainty of the model parameters in the development of a solution procedure for it.

8. ACKNOWLEDGMENTS
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9. REFERENCES


Table 1. Comparison of the Results of ETCH-H with those of SA-UFT for the First Scenario

<table>
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Table 2. Computational Results for Small Samples of Multi-Depot IRPs

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Table 3. Comparison of the Results of IRPB with those of IRP

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