A NOVEL FAULT ISOLATION AND PROGNOSTICS SCHEME FOR NONLINEAR DISCRETE-TIME SYSTEMS WITH PERFORMANCE GUARANTEES

Balajee T. Thumati
Electrical and Computer Engineering
btr74@mst.edu

J. Sarangapani
Electrical and Computer Engineering
sarangap@mst.edu

ABSTRACT

In this paper, novel fault isolation and prognostics (FIP) scheme is introduced for nonlinear discrete-time systems. In the FIP scheme, upon detecting a fault, a bank of fault isolation estimators is activated for identifying the faults. Asymptotic convergence of the isolation scheme and the type of faults that can be identified by the proposed design are discussed. A parameter-based prognostics scheme is then developed using the bank of fault isolation estimators in order to predict time-to-failure (TTF). Finally, an example is used to verify the effectiveness of the FIP scheme.

1. INTRODUCTION

In the last few decades, different analytical redundancy techniques were proposed to address fault diagnosis in complex systems [1, 2], which appear to be cost effective and easy to implement. In analytical redundancy techniques, a model representative of the system is used. A residual signal is generated by comparing the output of the system to that of the model. If the residual exceeds a predefined threshold, then a fault is declared active. Available methods on fault detection and diagnosis [1-4] use some sort of residual signal.

In particular, structured and fixed directional residuals are derived in [1] using an observer-based scheme for fault isolation. Others generate residuals using parity relations [2] and eigenstructure assignment [3]. However, these schemes are applicable for linear system. In addition, the prognostics component is not addressed. Recently, adaptive and online approximation-based fault isolation schemes for uncertain nonlinear continuous-time systems have been developed [4]. This scheme is demonstrated to render bounded residuals with satisfactory performance for fault isolation. Time-to-failure determination and prognostics are not discussed.

By contrast, in certain data driven time-to-failure (TTF) schemes [5] that address only prognostics have assumed to depend on a specific degradation model, which has been found to be limited to the system or material type under consideration. It is envisioned that the unified fault isolation and prognostics scheme would alert the system operator about an impeding fault, identify the root cause, and also provides the time to failure. These would render better maintenance practices and improved system usage. Hence, in this paper, unified fault isolation and prognostics framework is introduced for nonlinear discrete-time systems. Upon fault detection using an online estimator, a bank of fault isolation estimators is activated to identify the type of fault that has occurred. These isolation estimators would approximate the dynamics of each fault type. A fault can be isolated when the residual associated with the matched isolation estimator is below a predefined threshold whereas at least one of the components of the residuals associated with all other estimators exceeds their corresponding threshold. Both the selection of the fault isolation threshold and the fault isolability conditions are derived mathematically for discrete-time systems.

Subsequently, by using the parameter update law of the matched isolation estimator, a novel prognostics scheme is developed in order to predict the time-to-failure. The failure is defined as a threshold on the parameters beyond which it is considered unsafe to operate the system. The stability of the fault isolation scheme including the online parameter tuning scheme is shown to be asymptotically stable. Finally, a simulation example is used to illustrate the FIP scheme. Next, the system under investigation is introduced.

2. SYSTEM DESCRIPTION

Consider the following class of nonlinear discrete-time systems

\[ x(k+1) = \omega(x(k),u(k)) + \eta(x(k),u(k)) + \Gamma_1(k-k_1)p(x(k),u(k)) \]  

where \( x \in \mathbb{R}^n \) is the system state vector, \( u \in \mathbb{R}^m \) is the control input vector, \( \omega : \mathbb{R}^n \times \mathbb{R}^m \rightarrow \mathbb{R}^n, \) \( \eta : \mathbb{R}^n \times \mathbb{R}^m \rightarrow \mathbb{R}^n, \) \( h : \mathbb{R}^n \times \mathbb{R}^m \rightarrow \mathbb{R}^n \) are smooth vector fields. The term \( \omega(x(k), u(k)) \) represents the known nonlinear system dynamics whereas \( \eta(x(k), u(k)) \) denotes the system uncertainty. The function \( h(x(k), u(k)) \) represents a vector of possible faults that can occur and their associated dynamics referred to as fault function. There are \( N \) types of possible faults that can occur in the system and thus \( h(x(k), u(k)) \) represents a set of fault functions defined by

\[ h_f(x(k), u(k)) = \{ h^1(x(k), u(k)), h^2(x(k), u(k)), \ldots, h^N(x(k), u(k)) \} \]

Additionally, each fault function \( h^q(x(k), u(k)), q = 1, 2, \ldots, N, \) is defined as \( h^q(x(k), u(k)) = \begin{bmatrix} \phi^q_1(x(k), u(k)) & \cdots & \phi^q_{m^q}(x(k), u(k)) \end{bmatrix}^T \)

where \( \phi^q_i, i = 1, 2, \ldots, n, \) is an unknown \( s^q \)-dimensional parameter vector and \( \phi^q_i : \mathbb{R}^n \times \mathbb{R}^m \rightarrow \mathbb{R}^{s^q} \) is a known smooth vector field.

In the above definition, each fault vector is distinct, satisfies the linear in the unknown parameter assumption (LIP). Also, \( f^q_i \) represents the fault structure of the \( q^{th} \) fault affecting the \( f^q_i \).
state equation. In addition, the unknown parameter \( \sigma^q \) is the magnitude of the \( q \)-th fault affecting the \( i \)-th state equation. The time profile \( \pi(k - k_i) \) of the faults are modeled by

\[
\pi(k - k_i) = \text{diag}(\Omega_1(k - k_i), \Omega_2(k - k_i), \ldots, \Omega_n(k - k_i))
\]

where

\[
\Omega_i(\tau) = \begin{cases} 0, & \text{if } \tau < 0 \\ 1 - e^{-\kappa_i \tau}, & \text{if } \tau \geq 0 \end{cases} \quad \text{for } i = 1, 2, \ldots, n \tag{2}
\]

and \( \kappa_i > 0 \) is an unknown constant that represents the rate at which the fault in the corresponding state \( x_i \) occurs. The term \( \Omega_i(\tau) \) approaches a step function when \( \kappa_i \) is large, which in turn represents an abrupt fault.

**Remark 1:** Modeling faults using the above time profile is common in the fault detection literature [4], [7].

The following assumptions are required.

**Assumption 1:** The state and the input vectors are bounded prior to and after the fault occurrence. This is a standard assumption commonly found in the literature [4], [8]. In other words, there exist two compact sets \( \mathcal{X} \subset \mathbb{R}^n \), \( U \subset \mathbb{R}^m \), such that \( x(k) \in \mathcal{X} \) and \( u(k) \in U \) for all \( 0 \leq k \leq k_f \).

**Assumption 2:** The modeling uncertainty is unstructured and bounded [4], [8], i.e.,

\[
|\eta_i(z(k), u(k))| \leq \eta_{i,U}, \quad \forall (z, u) \in (\mathcal{X} \times U), \quad i = 1, 2, \ldots, n
\]

where \( \eta_{i,U} \geq 0 \) is a known constant.

In the next section, the fault detection scheme in discrete-time is revisited [8] prior to isolation discussion.

### 3. FAULT DETECTION SCHEME

Consider the nonlinear estimator described by

\[
\dot{x}(k + 1) = \dot{A} x(k) + \dot{B} u(k) + \dot{C} \sigma^q(\dot{x}(k), u(k)) - \dot{A}^x x(k) - F(x(k)) \tag{3}
\]

where \( \dot{x} \in \mathbb{R}^n \) is the estimated state vector, \( \dot{e}^d \in \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R}^{n \times n} \rightarrow \mathbb{R}^n \) is the online approximator in discrete-time (OLAD) that is used for fault detection, \( \dot{e}^d \in \mathbb{R}^{n \times n} \) is a set of adjustable parameters of the OLAD, \( \dot{A} = \text{diag}(\dot{A}_1, \dot{A}_2, \ldots, \dot{A}_n) \) is a diagonal matrix chosen by the user, and \( \dot{F}(x) \) is the robust term to be defined later. Prior to the occurrence of the fault, the initial values for the estimator (3) are taken as \( \dot{x}(0) = x(0), \dot{e}^d(0) = \dot{e}^d_0 \), such that \( \dot{h}(x, u, \dot{e}^d) = 0 \) for all \( x \in \mathcal{X} \) and \( u \in U \).

**Remark 2:** Upon fault detection, the OLAD and the robust terms are initiated. The OLAD approximates the unknown fault dynamics online whereas the robust term ensures that the residual generated using the estimator and actual system converges to zero asymptotically. The fault isolation scheme is also initiated upon fault detection.

Define the detection residual or state estimation error as

\[
e = x - \dot{x}.
\]

Then from (1) and (3) prior to the fault occurrence, the detection residual dynamics are defined as

\[
e(k + 1) = \dot{A}^e e(k) + \eta(x(k), u(k)) \tag{4}
\]

A fault is declared active when the detection residual exceeds a predefined threshold asserted via a dead-zone operator. Next, to tune the OLAD online, the following parameter update law is proposed as

\[
\dot{\gamma}^d(k + 1) = \dot{\gamma}^d(k) + \alpha \phi(k) \text{D} e(k)^T(k + 1)
\]

where \( \alpha > 0 \) is the learning rate and \( \phi(k) = \varphi(x(k), u(k)) \) is the basis function such as a RBF, sigmoid etc. Next, the robust term is defined as

\[
F(k) = \frac{\left(\dot{\gamma}^d(k)\right)^T b^d}{\left(b^d\right)^T \dot{\gamma}^d(k) b^d + c^d},
\]

where \( c^d > 0 \) is a user defined constant and \( b^d \in \mathbb{R}^p \) is a constant vector. This robust term ensures that the estimator accurately tracks the system states (1) even after the occurrence of a fault and in the presence of disturbances.

To guarantee the stability of the fault detection scheme, the following theorem demonstrates asymptotic stability.

**Theorem 1** [10]: Let the proposed estimator in (3) be used to monitor the system given by (1). Let the update law given in (6) be used for tuning the unknown parameters of the OLAD. In the presence of bounded uncertainties, the detection residual, \( e(k) \), and parameter estimation errors \( \dot{e}^d(k) \) are locally asymptotically stable.

**Remark 3:** The proposed detection scheme renders asymptotic stability in contrast with the previously reported fault detection schemes for both nonlinear continuous-time [4] and discrete-time systems [5] where a bounded stability is shown. As a consequence, fault isolation and prognostics can be accurately performed here as will be observed later.

### 4. FAULT ISOLATION SCHEME

To effectively isolate \( N \) faults online, \( N \) different fault isolation estimators are used, which as described by

\[
x^u(k + 1) = A_i^u x^u(k) + \varphi_i(x(k), u(k)) + \dot{h}_i(x(k), u(k)) - A_i^u x(k) - \dot{F}(x(k)) \tag{7}
\]

where

\[
\dot{h}_i(x(k), u(k)) = \begin{bmatrix} \dot{h}_1(x(k), u(k)) \\ \dot{h}_2(x(k), u(k)) \\ \vdots \\ \dot{h}_n(x(k), u(k)) \end{bmatrix} = \begin{bmatrix} \dot{A}_1^d(x(k), u(k)) \\ \dot{A}_2^d(x(k), u(k)) \\ \vdots \\ \dot{A}_n^d(x(k), u(k)) \end{bmatrix}
\]

and \( \dot{h}^q \in \mathbb{R}^{n \times q} \) for \( q = 1, 2, \ldots, N_i, \quad i = 1, 2, \ldots, n \) is the estimated fault parameter of the \( i \)-th state variable. Also, \( \dot{A}_i^d = \text{diag}(A_1^d, A_2^d, \ldots, A_n^d) \), where \( A_i^d \) is chosen such that all the poles are within the unit disc. Next, the robust vector \( \dot{v}_i(k) \) for the fault isolation estimators is defined as
where $\tilde{b}_1^q, \tilde{b}_2^q, \ldots, \tilde{b}_n^q$ for $q = 1, 2, \ldots, N$, are appropriate dimensioned constant vectors and $c_1, c_2, \ldots, c_N$ are user selected scalar constants. Next, define the $i$th fault isolation residual of the $q$th fault isolation estimator as 

\[ e_i^q(k) = x_i(k) - \tilde{x}_i^q(k) \]

Hence a fault is isolated if the $i$th isolation residual associated with the $q$th fault function is zero under ideal conditions or within its corresponding threshold, i.e., $|e_i^q(k)| \leq \beta_i^q(k)$ for general case. Consequently a set of adaptive thresholds $\{\beta_i^q(k), \forall i \in \{1, 2, \ldots, q\}, \forall k \}$ for the bank of fault isolation estimators is designed such that the faults in the system could be identified. The fact that none of the isolation estimators matches the fault that has occurred indicates that it is a new and unknown type of fault, and the approximated fault model from detection estimator can be used to create a new fault isolation estimator.

In order to tune the fault isolation estimators online, the following parameter update law is proposed

\[ \dot{\theta}_i^q(k + 1) = \theta_i^q(k) + \alpha_i^q f_i^q(x(k), u(k)) (x_i^q(k) - \tilde{x}_i^q(k)) \| x_i^q(k) - \tilde{x}_i^q(k) \|^2 \]

where $\alpha_i^q > 0$ is the learning rate, $\gamma_i^q > 0$ is the adaptation rate, and $f_i^q(x(k), u(k))$ is the known basis function.

**Remark 4:** This parameter update law is asymptotically stable provided the isolation errors are bounded. This property will help in prognostics as will be observed later. Moreover, the update law in (8) for fault isolation scheme is different than the one in (8) used for detection.

In the following theorem, the asymptotic convergence of the $i$th fault residual of a $q$th fault isolation estimator is presented. Next, following Lemma is needed.

**Lemma 1:** The term comprising of the system uncertainty $(\eta_1(x(k), u(k)), \ldots, \eta_N(x(k), u(k)))$, basis functions along with the ideal parameters of the estimators are bounded according to

\[ s_{i_1}^q(k) \leq \sum_{i_1}^n s_{i_1}^q(k) \leq \sum_{i_1}^n \beta_i^q(k) \| \dot{\theta}_i^q(k) \|^2 \leq \sum_{i_1}^n \beta_i^q(k) \| \dot{\theta}_i^q(k) \|^2 \]

where $\beta_i^q(k)$ is a non-linear function, $\beta_i^q(k), \beta_i^q(k), \beta_i^q(k)$, and $\beta_i^q$ are computable positive constants, and

\[ \sum_{i_1}^n \beta_i^q(k) \leq \alpha_i^q \gamma_i^q(k) \| \dot{\theta}_i^q(k) \|^2 \leq \alpha_i^q \gamma_i^q(k) \| \dot{\theta}_i^q(k) \|^2 \]

**Proof:** Use some standard norm inequalities. Assumption 1, and the fact that the uncertainties can be expanded as a function of the isolation residual and error in adaptive estimation parameters. The steps of this proof follow similar steps in proving the boundedness for a multi-layer neural network controller in continuous-time [9].

**Theorem 2 (Performance analysis):** Let the proposed fault isolation estimator in (7) be used to monitor the nonlinear discrete-time system in (1). Let the update law expressed in (8) be used for tuning the unknown parameters of each fault function or estimator. Then, in the presence of bounded uncertainties, the $i$th fault isolation residual of a $q$th fault isolation estimator, $e_i^q(k)$, and the $i$th parameter estimation error of a $q$th fault isolation estimator, $\tilde{e}_i^q(k)$, are locally asymptotically stable.

**Proof:** From (1) and (7), the fault residual dynamics are given by

\[ e_i^q(k + 1) = \dot{e}_i^q(k) + (1 - e_i^q(k-1)) (\dot{\theta}_i^q(k) - \theta_i^q(k)) \]

\[ f_i^q(x(k), u(k)) + \eta_i(x(k), u(k)) - \tilde{e}_i^q(k) \]

Substituting the robust term as

\[ c_i^q = \begin{pmatrix} \dot{\theta}_i^q(k) \\ \theta_i^q(k) \end{pmatrix} \]

 subtracting \[ \sum_{i_1}^n \beta_i^q(k) \leq \alpha_i^q \gamma_i^q(k) \| \dot{\theta}_i^q(k) \|^2 \]

where $c_i^q$ is a constant vector, in the above equation, to get

\[ e_i^q(k + 1) = \dot{e}_i^q(k) + (1 - e_i^q(k-1)) (\dot{\theta}_i^q(k) - \theta_i^q(k)) \]

\[ f_i^q(x(k), u(k)) + \eta_i(x(k), u(k)) + \tilde{e}_i^q(k) \]

\[ \sum_{i_1}^n \beta_i^q(k) \leq \alpha_i^q \gamma_i^q(k) \| \dot{\theta}_i^q(k) \|^2 \]

Next, consider a Lyapunov function as

\[ V_i^q = \| e_i^q(k) \|^2 + \frac{1}{\alpha_i^q} \text{tr} \left( \dot{e}_i^q(k)^T \dot{e}_i^q(k) \right) \]

whose first difference is given by

\[ \sum_{i_1}^n \beta_i^q(k) \leq \alpha_i^q \gamma_i^q(k) \| \dot{\theta}_i^q(k) \|^2 \]

\[ \sum_{i_1}^n \beta_i^q(k) \leq \alpha_i^q \gamma_i^q(k) \| \dot{\theta}_i^q(k) \|^2 \]

\[ \sum_{i_1}^n \beta_i^q(k) \leq \alpha_i^q \gamma_i^q(k) \| \dot{\theta}_i^q(k) \|^2 \]
\[ \begin{align*}
\Delta V_q & = -\frac{(\tilde{e}_q(k+1)^2)}{(\tilde{e}_q(k)^2)} - \frac{1}{\alpha_q^2} \left( \frac{\bar{e}_q(k+1)}{\bar{e}_q(k)} \right)^2 \frac{\tilde{e}_q(k+1) \tilde{e}_q(k)}{\bar{e}_q(k)} \right) \end{align*} \] (11)

Substituting (10) in \( \Delta V_q \) of (11), and substitute (8) in \( \Delta V_2 \) of (11), performing some mathematical manipulation, and taking Frobenious norm, the overall first difference is expressed as

\[ \Delta V_q \leq - (1 - \nu \tilde{A}^T(\tilde{e}_q(k))) \left[ (\tilde{e}_q(k))^2 \right] - \left( \tilde{f}_q^a (x(k), u(k)) + \eta_q (x(k), u(k)) - \tilde{f}_q (k) \right) \] (13)

Since from Theorem 2, the stability of the update law in (8) is guaranteed, hence the parameter estimation error could be taken to be bounded as

\[ \left( \tilde{f}_q^a (x(k), u(k)) \right) \right) \leq \left( \tilde{f}_q^a (x(k), u(k)) \right) \] (13)

Hence \( \Delta V_q < 0 \) in (12) provided

\[ \begin{align*}
(c_q)_{\min} & = \left( \frac{\tilde{f}_q^a (x(k), u(k))}{\eta_q (x(k), u(k))} \right) \right) \leq \frac{(1 - \beta_q^a - \beta_q^r) \tilde{f}_q^a (x(k), u(k))}{4 - \nu \tilde{A}^T(\tilde{e}_q(k)) \left[ (\tilde{e}_q(k))^2 \right] - \left( \tilde{f}_q^a (x(k), u(k)) + \eta_q (x(k), u(k)) - \tilde{f}_q (k) \right) \right) \] (13)

Hence if (14) is satisfied, then the \( q \) th fault would be isolated. As such (14) is not realizable since few of the terms in the right hand-side of the above equation is not known, thus by replacing the unknown terms with known values, the definition in (14) could also be used as a fault isolation threshold, i.e., take \( \kappa_{i_0} > \kappa_i \) as a constant and since \( \theta < 1 - e^{-\kappa_i (k-h)} < 1 \), thus the adaptive threshold is defined by

\[ \begin{align*}
(\tilde{f}_q^a (x(k), u(k)) \right) \right) \leq \frac{(1 - \beta_q^a - \beta_q^r) \tilde{f}_q^a (x(k), u(k))}{4 - \nu \tilde{A}^T(\tilde{e}_q(k)) \left[ (\tilde{e}_q(k))^2 \right] - \left( \tilde{f}_q^a (x(k), u(k)) + \eta_q (x(k), u(k)) - \tilde{f}_q (k) \right) \right) \] (13)

The first difference, \( \Delta V_q < 0 \) in (12), which shows stability in the sense of Lyapunov provides the gains are selected above. Thus the fault isolation residual, \( \tilde{e}_q(k) \), and the parameter estimation errors, \( \tilde{f}_q^a (k) \), approach zero as \( k \to \infty \) [10]. For an ideal case scenario, i.e., system without uncertainty, there is no need of a robust term in the fault isolation estimator and still asymptotic convergence could be proven.

Next, the following theorem is introduced to illustrate the isolation of several fault types in a system.

**Theorem 3 (Fault Isolation):** If the incipient fault \( q \) occurs, there exist some finite time \( k > k_q \), and \( i \in \{1, 2, \ldots, n\} \), such that the \( \tilde{f} \) th component of the \( q \) th estimator satisfies \( \tilde{e}_q^i (k) \leq \beta_q^i (k) \), and then the occurrence of the \( q \) th fault could be concluded.

**Proof:** For the incipient fault \( q \) occurs, using (1) and (7), the \( \tilde{f} \) th component of the fault isolation residual is defined by

\[ \tilde{f}_q^i (x(k), u(k)) + \eta_i (x(k), u(k)) - \tilde{f}_q^i (k) \] (13)

where \( \tilde{f}_q^i (k) \) is a known time dependent function and using the maximum values. Taking absolute value, thus (13) could be rewritten as

\[ \left| \tilde{f}_q^i (k) \right| \leq \left( \tilde{f}_q^i (k) \right) \right) \leq \frac{(1 - \beta_q^a - \beta_q^r) \tilde{f}_q^a (x(k), u(k))}{4 - \nu \tilde{A}^T(\tilde{e}_q(k)) \left[ (\tilde{e}_q(k))^2 \right] - \left( \tilde{f}_q^a (x(k), u(k)) + \eta_q (x(k), u(k)) - \tilde{f}_q (k) \right) \right) \] (13)

Hence if (14) is satisfied, then the \( q \) th fault would be isolated.

**Theorem 4 (Fault Isolability):** Let the fault isolation
estimation scheme described in (7) be used for isolating faults online. Then an incipient $q^{th}$ fault would be isolated if for each $r \in \{1, \ldots, N\} \setminus \{q\}$, the following condition is satisfied:

$$
\sum_{j=1}^{k} (\alpha_j) \left[ \left[ 1 - e^{-\beta_j (J_{r})} \right] \left( \theta^{(r)} \right) \right] - \left( \theta^{(j)} \right) = \left( \theta^{(j)} \right)_{max} = 0
$$

$z \in \left( \alpha_j \right) [\theta_j, \theta_j] + \sum_{j \neq k} \left( \alpha_j \right) e^{-\beta_j (J_{r})} \sum_{j \neq k} \left( \alpha_j \right) \left( \theta^{(j)} \right) + \left( \theta^{(j)} \right)_{max} = 0

f_j^{(r)} = \omega \left( x, u, \eta_{j} \right) + \sum_{j=1}^{k} \left( \alpha_j \right) e^{-\beta_j} + \sum_{j=1}^{k} \left( \alpha_j \right) \left( \theta^{(j)} \right)

+ \sum_{j=1}^{k} \left( \alpha_j \right) \left( \theta^{(j)} \right)_{max} (x, u, \eta_{j})

(16)

**Proof:** The details of the proof are omitted due to space considerations. If the condition in (16) is satisfied, then $\left( \theta^{(j)} \right) > \beta_j (k)$. This implies that the occurrence of the $r^{th}$ fault is isolated. If this condition is satisfied, then for each $r \in \{1, \ldots, N\} \setminus \{q\}$, the $q^{th}$ fault could be isolated.

The last two theorems demonstrate the performance of the fault isolation estimation scheme provided that fault can be separated. In the next section, the prognostics scheme is introduced.

4. PROGNOSTICS SCHEME

The prognostics scheme is derived by using the upper bound of the isolated fault parameter described in the previous section. This utilized to predict the TTF by projecting them at each time instant to their corresponding failure thresholds. To estimate TTF, an explicit mathematical equation is derived, which is based on the online parameter update law presented in (8). This equation is then developed to use an algorithm for the continuous prediction of TTF at every time instant as given next.

**Theorem 5 (Time to Failure Determination):** In the presence of a fault, the TTF for the $j^{th}$ fault parameter, where $j = 1, 2, \ldots, q, q = 1, 2, \ldots, N$, $l = 1, 2, \ldots, n$, at the $k^{th}$ time instant can be determined using

$$
k_{j}^{(k)} = \frac{\bar{\theta}_j}{\bar{\alpha}_j (\theta_j^{max} - \theta_j^{(k)})}

+ k_{0j}

(17)

$$
where

$$
\theta_j^{max} = \left( \theta_j \right)_{max} = \left( \theta_j \right)_{0} = \left( \theta_j \right)_{0} = \left( \theta_j \right)_{0}

$$

$k_{0j}$ is the estimated TTF, $k_{0j}$ the time instant when the prediction starts (bearing in mind that $k_{0}$ is the fault detection time or initial value which increases incrementally), $(\theta_j^{max})$ is the maximum value of the fault parameter, and $(\theta_j^{(k)})$ is the value of the fault parameter at the time instant $k_{j}^{(k)}$.

**Remark 5:** The mathematical equation (17) is derived for the $j^{th}$ fault parameter. In general, for a given system with $N$ possible faults, the TTF would be $k_{j}^{(k)} = \min(k_{j}^{(k)})$. $j = 1, 2, \ldots, q, q = 1, 2, \ldots, N$, $l = 1, 2, \ldots, n$, where $s_l$ are the number of parameters of the different fault vectors. The TTF is the time elapsed when the first parameter reaches its limit.

**Proof:** Proof omitted due to space considerations.

The matched isolation estimator provides the root cause whereas the TTF indicates when an impending failure will occur. These two together will constitute prognostics.

In the next section, a simulation example is used to illustrate the proposed FIP scheme.

5. SIMULATION RESULTS

Consider the following second order discrete-time representation of a jet engine compression system with no stall described by

$$
x_j^{(k+1)} = \Delta t(-x_j^{(k)} - 1.5x_j^{(k)} - 0.5x_j^{(k)}) + x_j^{(k)}

x_j^{(k+1)} = \Delta t(-u(k) + \eta(x(k), u(k))) + x_j^{(k)}

(18)

$$
where $x_j$ and $x_j$ are the states of the system. We consider two type of faults defined by $h^{1}(x_j^{(k)}) = \theta_1 \omega(0,3x_j^{(k)})$ and $h^{2}(x_j^{(k)}) = \theta_2 x_j^{2}(k)$. The faults are assumed to occur only in the system state $x_j$. A bank of two fault isolation estimators are designed for identification of the faults in (18) as

$$
x_j^{1}(k+1) = \Delta t(-x_j^{(k)} - 1.5x_j^{(k)} - 0.5x_j^{(k)}) + x_j^{(k)} + 0.15(x_j^{(k)} - x_j^{1}(k))

+ \theta_1 (x_j^{2}(k))_{+}

(19)

$$
x_j^{2}(k+1) = \Delta t(-u(k)) + x_j^{(k)} + 0.01(x_j^{(k)} - x_j^{2}(k))

+ \theta_2 x_j^{2}(k)

(20)

For this simulation, the fault parameters are taken as $\theta_1 = 3, \theta_2 = 0.4$; and $\Delta t = 1$ sec, $\eta(x(k), u(k)) = 0.5$ for $\Delta t < 10$ sec, else zero. Note due to space constraints not all the values of the parameters used in the simulation have been included. In this simulation, we assume the fault type 1, i.e., $h^{1}(x_j^{(k)})$ to occur at the 15th second of system operation. The simulation results are presented in Figs. 2 through 5. In Fig. 2, the online estimation
of the fault parameter is compared against the actual value of the fault parameter. The performance appears to be highly satisfactory.

![Graph](image1)

**Fig 2:** Actual and estimated fault parameter.

![Graph](image2)

**Fig 3:** Fault residual of the 1st isolation estimator and its threshold.

![Graph](image3)

**Fig 4:** Fault residual of the 2nd isolation estimator and its threshold.

![Graph](image4)

**Fig 5:** TTF determination after the detection of the fault.

Subsequently, the banks of fault isolation estimators ([24]-[25]) are initiated to isolate the fault in the system (23). Figures 3 and 4 are used to isolate the fault type 1. As observed in Fig. 3, since the first isolation estimator matches with the fault, the fault isolation residual remains with its corresponding threshold. By contrast, as expected, the isolation residual of the second fault isolation estimator exceeds its respective threshold. Hence the system renders a decision that fault type 1 has occurred.

Next for this fault type, the TTF is determined based on the online estimate of the fault parameter \( \delta(t) \). This TTF value is shown in Fig. 5, where the accuracy of the prediction improves as the fault isolation scheme learns the change in the system dynamics better. Thus this simulation example illustrates the fault detection, fault isolation and prognostics proposed in this paper.

6. CONCLUSIONS

In this paper, a novel FIP scheme was developed. Extensive mathematical results on the stability and the conditions for fault isolation were derived. Finally, a simulation example shows that the FIP scheme successfully isolates the fault that has occurred in the system. However, the proposed FIP scheme requires all state measurability, which will be relaxed as part of the future work.

7. ACKNOWLEDGMENTS

The authors would like thank Intelligent Systems Center and NSF I/UCRC on Intelligent Maintenance Systems for supporting this research work.

8. REFERENCES


