ABSTRACT
One of the major threats to Aviation industry is the in-flight impact of birds. Aircraft windshields are intensively vulnerable to damage and hence need a certification requirement for a proven level of impact resistance. Bird strike experiments are very expensive and henceforth explicit numerical modeling techniques have grown importance. This compilation mainly relies on the theory of hydrodynamic impact and addresses basic shockwave equations. A smooth particle hydrodynamics based approach is chosen and the numerical simulation is carried out using contact-impact coupling algorithm. The simulation is run using the nonlinear explicit finite element code LS-DYNA ver.971, developed by Lawrence Livermore National Laboratories. A traditional design of experiments approach, factorial design, has been utilized to facilitate the economical prediction of factors significantly affecting the pressure response in the bird strike analysis. Impact velocity, bird mass, bird aspect ratio, porosity and obliquity of impact are the various parameters investigated in the present work. The results show that main effects velocity and obliquity of impact, and ‘porosity x obliquity’ interaction effect, are the factors significantly affecting the pressure response in a bird strike analysis.

1. INTRODUCTION
Bird strikes have been a significant safety threat and unavoidable menaces to both civil and military aircrafts. The bird strike incident can have tragic consequences, especially for small generation aviation airplanes. Bird strikes cause significant economic loss to the aviation industry, estimated over $1.2 billion annually [1]. These incidents are critical during takeoff and landing phases. This is one reason for limiting the maximum airplane speeds to 250kt below 10000 ft altitudes [2].

The forward-facing components of an aircraft such as windshields, leading edge structures, aircraft engines etc are often susceptible to such strikes. The U.S. Federal Aviation Administration (FAA) and European Joint Aviation Authority (JAA) have set certification standards which require an aircraft be designed to successfully complete a flight after an impact with a standard-size bird [2]. Further testing with higher impact velocities is carried out to determine the actual limits of the structure. The use of real birds lends realism to the test results but complicates the testing procedure, very costly and time consuming.

The number of variables, the bird strike analysis deals with, is high. This makes the bird strike analysis complex. The various parameters include bird density, velocity and mass of bird, bird material configuration, high strain rates and contact properties. Most of the models, initially developed, used force-impulse equation. Semi-empirical equations were developed to simulate the entire impact event. However, in most of the cases, these models failed to predict the damage to its details. They failed to account for the complex interactions during the impact.

Finite element analysis is capable of accurately simulating the bird impact, and thus the geometry of the impacted structure. The finite element method is good at simulating the transient loading of bird on the target. Because, the bird strike analysis deals with high strain rates, nonlinearity and large deformation accompany the impact. Finite element analysis is proved to deal with nonlinearities in structures and has the capability of predicting the penetration of bird following the initial hit.

The nonlinear explicit finite element code LS-DYNA has been used by many researchers, especially in high velocity impacts. In the current investigation, LSTC LS-DYNA ver.971 will be used.

The use of real birds has many drawbacks. One of the major drawbacks is the lack of repeatability. The impact loads vary from test to test. Testing with real birds also has sanitary and aesthetic disadvantages. Explicit numerical techniques have grown importance and are a viable alternative to several extensive and difficult experimental iterations of the test structure.
A bird strike incident is characterized by high impact loads and short duration. The impact duration is typically in the range of milliseconds. An important step in the bird strike simulation is the development of an “artificial bird model.” The artificial bird models would replace real birds in actual impact tests. The accurate modeling of an artificial bird includes the shape and material modeling of the bird.

Design of Experiments (DOE) is a systematic and rigorous methodology to solving engineering problems. DOE is the simultaneous study of several variables. Combining several variables in one study, the amount of simulation experimentation is dramatically reduced and yields greater knowledge. DOE is a viable tool for gaining knowledge through experimentation; increases possibility to extract large amount of information in a matrix, from a small fraction of data. A Design of Experiments (DOE) approach, factorial design, is used to statistically evaluate the significant factors and their interactions affecting the bird impact pressures.

2. LITERATURE REVIEW

Bird strike analysis is not a new problem and relatively good amount of work has been done. Barber et al. [3, 4] and Wilbeck et al. [5-7] conducted extensive experimentation on bird strike analysis and concluded the fluid like behavior of birds upon impact. Cassenti [8] developed the governing equations for the hugoniot pressure. Wilbeck [5] developed an empirical bird model based on the hydrodynamic theory and compared it with the experimental results. The empirical model was in good agreement with the experimental results for medium sized birds. Wilbeck also proposed equation of state [EOS] models for hugoniot shock phase and steady state phase. McCallum et al. [9] worked on using multi-material bird models. McCallum also worked on different bird geometries for the impact analysis. Huertas [10] and Goyal et al. [11] carried out a computational study on bird impact phenomenon using lagrangian, Arbitrary Lagrangian Eulerian (ALE) and Smooth Particle Hydrodynamics (SPH) formulations. Nizampatnam et al. [12, 13] worked extensively on multi-material heterogeneous bird models and equation of state [EOS] models, and made significant contribution.

Not much work has been presented on investigating the behavior of various parameters of the bird. The study is limited and is done using the traditional one-factor-at-a-time (OFAT) experiments (i.e. simulation experiments). The one-factor-at-a-time (OFAT) experiments, however, can only estimate the individual behavior of factors; never the interaction. With the advancement of statistical Design of Experiments (DOE) approach, the interaction effects can also be explored. The present compilation uses factorial design of experiments methodology to conclude the terms significant to pressure response in a bird strike analysis.

3. FINITE ELEMENT MODELING AND VALIDATION

In this section, a brief introduction to hydrodynamic theory is provided. The various phases in a bird strike analysis, Equation of State models are discussed. At the end, the finite element models considered for the bird strike analysis and the validation of simulation results with Wilbeck [5] analytical and experimental results are presented.

3.1. Hydrodynamic Theory

The theory of Hydrodynamics can be used to study high velocity impacts. The hydrodynamic theory is based on the belief that during impact the projectile material tends to behave as a fluid. The requirement for a material to flow is that the stresses generated during impact should exceed the strength of the material. The material strength and viscosity are neglected and a simple pressure-density-energy equation of state is used to describe the material behavior. In the Hydrodynamic approach, it is the material density which dominates the strength. This theory assumes that in a high velocity impact, the projectile material would behave as a fluid with same material density [5].

3.2. Equation of State Models for Bird

An Equation of State (EOS) is a relation between state variables. It is a thermodynamic equation that describes the state of matter under a given set of physical conditions. It is a constitutive equation that provides mathematical relationship between two or more state functions such as temperature, volume, pressure and internal energy.

\[ P = P(V, E) = P(V, T) \]  \hspace{1cm} (1)

where \( V \), \( E \) and \( T \) are volume, internal energy and temperature respectively. Wilbeck [5] carried out extensive amount of research in finding out precise Equation of State models for the shock phase and steady state phase. The Equation of State model formulation for the Hugoniot phase is presented below.

The shock wave progression and the condition ahead of and behind a shock moving through a material can be described by the following conservation laws:

Conservation of mass

\[ \rho_1 u_s = \rho_2 (u_s - u_p) \]  \hspace{1cm} (2)

Conservation of momentum

\[ P_1 + \rho_1 u_s^2 = P_2 + \rho_2 (u_s - u_p)^2 \] \hspace{1cm} (3)

Conservation of Energy

\[ P_2 u_p = \rho_1 u_s (E_2 - E_1 + \frac{u_p^2}{2}) \] \hspace{1cm} (4)

where \( u_s \) is the shock velocity, \( u_p \) is the translational particle velocity or simply projectile velocity, \( \rho_1 \) and \( \rho_2 \) are the respective densities of the material in front of the shock and behind it, \( E_1 \) and \( E_2 \) are the respective energies before and after compression, \( P_1 \) and \( P_2 \) are the pressures in unshocked and shocked regions respectively. Equations (2) and (3) can be combined to obtain Hugoniot pressure \( P_H \) as below:

\[ P_H = P_2 - P_1 = \rho_1 u_s u_p \] \hspace{1cm} (5)

From the above it is clear that the Hugoniot pressure is a function of initial projectile density, shock velocity and the projectile velocity. The shock velocity is unknown. For most solids and liquids, the shock velocity and particle velocity are related by the expression:

\[ u_s = c_0 + ku_p \] \hspace{1cm} (6)
where \( c_0 \) is isentropic wave speed and \( k \) is a constant. The above equation is called “linear Hugoniot.”

If the bird is assumed to behave as a fluid and take the properties of water, the above relation takes the form:

\[
U_{s,\text{water}} = c_{0,\text{water}} + k_{\text{water}} u_p
\]  

(7)

where \( c_{0,\text{water}} \) is the velocity of sound in water, 1482.9 m/s and \( k_{\text{water}} = 2.0 \).

From Equation (2) the continuity equation can be written as:

\[
\frac{u_p}{u_s} = 1 - \frac{\rho_1}{\rho_2} = q
\]  

(8)

where the subscripts 1 and 2 refer to the regions in front of and behind it.

Substituting Equation (2) into Equation (6) and rearranging:

\[
\frac{u_s}{c_0} = \frac{1}{1 - kq}
\]  

(9)

Substituting Equations (8) and (9) into Equation (5) gives:

\[
P_H = \frac{\rho_1 c_0^2 q}{(1 - kq)^2}
\]  

(10)

The pressure-density relationship for a one-dimensional shock developed by Cogolev takes the form:

\[
P_H = A \left( \frac{\rho_2}{\rho_1} \right)^B - 1
\]  

(11)

where A and B are material constants. Ruoff approximated the constants by the following expressions:

\[
A = \frac{\rho_1 c_0^2}{(4k - 1)}
\]  

(12)

\[
B = (4k - 1)
\]  

(13)

where \( k \) is a constant.

The Equation (11) can be re-arranged to obtain:

\[
\frac{\rho_1}{\rho_2} = (\frac{P_H}{A} + 1)^{-\frac{1}{B}}
\]  

(14)

All the real birds contain certain quantities of entrapped air. A homogeneous mixture of water and air was believed to behave as a real bird. When porosity effects are included, the linear Hugoniot no longer holds good for the relation becomes nonlinear. Wilbeck [5] used mixture theory to develop an Equation of State for the porous material.

\[
\left( \frac{\rho_1}{\rho_2} \right)_{\text{porous}} = (1 - z) \left( \frac{\rho_1}{\rho_{2,\text{water}}} \right) + z \left( \frac{\rho_1}{\rho_{2,\text{air}}} \right)
\]  

(15)

where \( z \) is percentage by volume of air.

The density ratio for water can be obtained from Equations (2) and (7). The Equations (8) and (10) can be used for air. So Equation (15) takes the form:

\[
\left( \frac{\rho_1}{\rho_2} \right)_{\text{porous}} = (1 - z) \left( \frac{P_H}{A} + 1 \right)^{-\frac{1}{B}} + z(1 - q)
\]  

(16)

Equation (16) has to be solved for the Hugoniot pressure generated during the impact.

3.4. Finite Element Models

In this section, three different bird shapes are assumed, and validated with Wilbeck analytical model and experimental results [5]. The numerical simulation is carried out using the Smooth Particle Hydrodynamics (SPH).

The smooth particle hydrodynamics was initially developed to investigate various astrophysical problems, and later extended to the field of computational fluid dynamics. This phenomenon is now being explored for its use in continuum mechanics. The main difference between the lagrangian and smooth particle hydrodynamics theories lies in the discretization of the continuum [14]. Smooth particle hydrodynamics is a meshless method. The lagrangian mesh is replaced with particles at the nodes. These particles are not directly connected to each other; the kernel function is the core of the method. Because the particles are not directly connected, this method suits well the bird impact phenomenon, large mesh distortions are not observed as opposed to the conventional lagrangian formulation. Throughout this compilation, the simulations are run using the smooth particle hydrodynamics in LS-DYNA.

![Figure 3 Simulation of cylindrical bird model at various time histories](image1)

![Figure 4 Simulation of ‘cylinder with hemispherical ends’ bird model at various time histories](image2)

![Figure 5 Simulation of ellipsoidal bird model at various time histories](image3)
Three different bird geometries—cylindrical, cylinder with hemispherical ends, and ellipsoidal—are considered for the finite element bird validation. Majority of the birds responsible for damage in a bird strike are medium sized. And the maximum airplane speeds are limited to 250 kt below 10,000 ft altitudes [2]. The same velocity can be assumed for the bird. The SI unit equivalent is 128.6 m/s. Bird mass is fixed at 2 kg and an aspect ratio of 2 is assumed. The plate is considered to be rigid, to evaluate the bird model alone and neglecting the effect of target on the bird. Figures 3-5 show the simulation of finite element bird models at different time periods.

The peak pressures, Hugoniot pressures, are extracted from the simulation results and plotted on a graph alongside Wilbeck analytical model and experimental results. Figure 6 shows the graph plotted with these results. Also simulations corresponding to the velocities 150 m/s, 200 m/s and 250 m/s, have been presented to validate the finite element model with the analytical model. It is clearly observed that the cylindrical model with hemispherical ends predicted the analytical model well. The cylindrical model with hemispherical ends underestimated the result by only 7.73% (velocity at 250 kt [2] or 128.6 m/s). The cylindrical model had an error of 9.61% and the ellipsoidal model over-estimated the pressure by 25.97%. This large difference could be due to low contact area in case of ellipsoidal model.

The artificial bird model predicted the pressure well; a parametric evaluation of this model using the Design of Experiments (DOE) approach can provide the relative significance of each of the parameters, which is very important.

4. DESIGN OF EXPERIMENTS
Design of experiments is a discipline that has very broad application. It is a systematic and rigorous methodology to solving engineering problems; applies principles and techniques at data collection stage, ensuring the generation of valid, defensible and convincing inferences [15]. It is an efficient and cost-effective way to model and analyze the relationships that describe bird impact pressure variations.

Typical one-factor-at-a-time (OFAT) experiments limit knowledge and can be a waste of time. In this context, the term experiments can be referred to simulation experiments. Design of Experiments (DOE) is the simultaneous study of several variables. Combining several variables in one study, the amount of simulation experimentation is dramatically reduced and yields greater knowledge. The major disadvantage of the one-factor-at-a-time strategy is that it fails to consider any possible interaction between the factors. Design of experiments is a viable tool for gaining knowledge through experimentation; increases possibility to extract large amount of information in a matrix, from a small fraction of data.

Design of experiments approach has several advantages over traditional one-factor-at-a-time experimentation. Design of experiments requires far fewer tests for validating results, tending towards significant cost and time savings. It can detect the interactions between variables that cannot be revealed when varying one factor at a time. These interactions are significant and often prove to be the key to break through improvements.

Many experiments involve the investigation of two or more factors. For these experiments, factorial designs are very efficient [15]. The most common case of these factorial designs involves ‘k’ factors each at two levels, generally referred to as ‘2k factorial designs’. These designs are widely used in ‘factor screening experiments’. Figure 7 contrasts a 23 factorial design with a one-factor-at-a-time experiment of equivalent replication [16].

![Figure 7 Comparison of two-level factorial designs (left) versus OFAT (right)](https://via.placeholder.com/150)

The two-level factorial design with three factors requires eight runs in total, whereas a one-factor-at-a-time experiment for the same requires sixteen runs. Due to its efficient parallel processing, the factorial design trumps the serial one-factor-at-a-time methodology; the efficiency of the factorial design becomes more pronounced as the number of factors (k) increase.

Designing a priori a numerical experiment is an important step in the factorial design- design of experiments approach. The experimental design mainly consists of selecting the
factors and their levels that will provide most of the information on the input-output relationship of a model in the presence of variation [16]. Five inputs (factors) have been considered responsible to pressures in a bird impact process. Different factorial level combinations are selected to ascertain the relative importance of each of these factors. The response variable of interest is the higouriot pressure generated during the impact. Table 1 lists the factors and their levels under investigation, and the response variable of interest.

### Table 1 Factors, level ranges, and response variables

<table>
<thead>
<tr>
<th>Factors</th>
<th>Label</th>
<th>Units</th>
<th>Levels</th>
</tr>
</thead>
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<tr>
<td></td>
<td></td>
<td></td>
<td>Level 1</td>
</tr>
<tr>
<td>Mass</td>
<td>X1</td>
<td>kg</td>
<td>1.5</td>
</tr>
<tr>
<td>Velocity</td>
<td>X2</td>
<td>m/s</td>
<td>150</td>
</tr>
<tr>
<td>Porosity</td>
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<td>(kg/m³)/(kg/m³)</td>
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</tr>
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<td>Aspect Ratio</td>
<td>X4</td>
<td>m/m</td>
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</tr>
<tr>
<td>Obliquity</td>
<td>X5</td>
<td>degrees (angle)</td>
<td>45</td>
</tr>
<tr>
<td>Response</td>
<td>Label</td>
<td>Units</td>
<td></td>
</tr>
<tr>
<td>Pressure</td>
<td>Y1</td>
<td>MPa</td>
<td></td>
</tr>
</tbody>
</table>

The analysis is based on a 25 full-factorial experiment design. Running the full matrix of all possible factor combinations implies estimating all main and interaction effects [15]. The present design requires a total of 32 simulation experiments to be run. Table 2 lists the various input factor combinations and the corresponding pressure output.

With a full-factorial experiment, a model containing five main effect terms, ten 2-factor interaction terms, ten 3-factor interaction terms, five 4-factor interaction terms, a 5-factor interaction term and a mean term, can be fitted. However, the present design has been limited to 2-factor interaction terms only. The higher order interaction terms have been omitted assuming that they are insignificant.

A statistical hypothesis testing procedure called analysis of variance (ANOVA) is used to test the significance of the data. The analysis is carried out in SAS v9.2; the obtained analysis of variance results are shown in Table 3. The model accounts for most of the variability in the response, achieving an adjusted R2 of 0.8367. The R2 is found to be 0.9157; it measures the proportion of total variability explained by the model. However, the R2 statistic always increases as factors are added to the model, even if the factors are not significant. In view of this, the R2-adjusted is considered in majority of scenario. The adjusted R2 statistic is a statistic that is adjusted for the size of the model; it can actually decrease if non-significant terms are added to model [15].

The p-value of 0.0058 corresponding to ‘porosity x obliquity’ interaction effect implies the conclusion that the interaction effect ‘porosity x obliquity’ has significant effect on the pressure response is made with 99.42% confidence. Or in simple lines, we say the porosity and obliquity interaction terms are significantly affecting the pressure response and need to be considered while modeling the phenomenon.

### Table 2 Design of experiment matrix listing the factorial combinations and their responses

<table>
<thead>
<tr>
<th>Exp #</th>
<th>Factorial design</th>
<th>Response</th>
<th>Exp #</th>
<th>Factorial design</th>
<th>Response</th>
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<tr>
<td></td>
<td>X1 X2 X3 X4 X5</td>
<td>Y1</td>
<td></td>
<td>X1 X2 X3 X4 X5</td>
<td>Y1</td>
</tr>
<tr>
<td>1</td>
<td>1.5 150 0.1 1 45</td>
<td>138.15</td>
<td>17</td>
<td>1.5 150 0.1 1 90</td>
<td>330.53</td>
</tr>
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<td>18</td>
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<td>6</td>
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<td>469.89</td>
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</table>

The similar conclusions are made on the main effects that are significantly affecting the pressure response. Because, the porosity and obliquity interaction terms are significant, we need not look into porosity and obliquity main effects. The main effects other than porosity and obliquity are looked into and the conclusion velocity main effect has significant influence on the pressure response is made with 99.99% confidence. The higher value of p can be explained in similar lines too. For example, the main effect, aspect ratio, has a p-value of 0.5161. This value of p can be explained in similar lines too. For example, the pressure response, is rejected with 48.39% confidence. Or in simple lines, we say the porosity and obliquity interaction terms are significantly affecting the pressure response and need to be considered while modeling the phenomenon.
5. CONCLUSIONS

Bird strikes are often unavoidable and can have tragic consequences to both civil and military aircrafts. The design of structural components against bird strikes requires indepth understanding of the bird strike phenomenon, which is quite complex in view of the convoluted and uneconomical experimental investigations. Explicit finite element analyses have grown importance and have become a viable alternative to development of artificial bird models. The finite element (FE) and design of experiments (DOE) methods are used in tandem to obtain a better understanding of the bird strike phenomenon. Finite element models with different geometries are compared to the experiment results and the empirical model developed by Wilbeck. Having validated the artificial bird model, factorial design of experiments approach is used, as opposed to one-factor-at-a-time (OFAT) experiments, to carry out parametric evaluation. The model signatures indicate that the velocity and obliquity main effects and ‘porosity x obliquity’ interaction effects are significant terms affecting the pressure response in a bird impact phenomenon. The present work can be furthered to bird strike on windshields with various material combinations and optimize the structural component design using response surface methodologies (RSM).

6. ACKNOWLEDGMENTS

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7. REFERENCES